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## Role of interactions in layered nanorings

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**Abstract:** The magnetic properties of nanorings consisting of alternate layers of magnetic and non-magnetic materials are studied. The relative stability of the different magnetic configurations is examined as a function of the ring geometry. The interactions between the magnetic layers play an important role which is evidenced in the phase diagrams presented.

**Keywords:** nanomagnetism; magnetic rings; layered structures; magnetic phase diagrams.

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## 1 Introduction

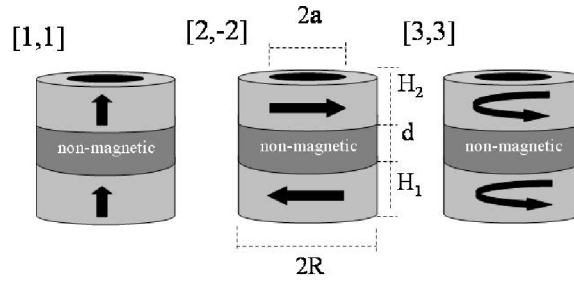
Over the last decade, considerable effort has been directed to the investigation of magnetic properties of nanoparticles. Different geometries have been considered, including magnetic wires, tubes, rings and dots. In particular, multilayer stacks of ring-shaped elements have been considered as potential building blocks for magnetoresistive random access memory [1,2]. Interest has been focused on the determination of the magnetic configuration of such layered nanorings as a function of their geometry, having in mind their great potential for applications.

Geometrically, ring-shaped particles are characterised by their external and internal radii,  $R$  and  $a$ , respectively, and height,  $H$ . Magnetic measurements as well as micromagnetic simulations on such systems have identified three characteristic ideal internal magnetic configurations, namely the out-of-plane ferromagnetic ( $c = 1$ ), the onion ( $c = 2$ ), and the vortex ( $c = 3$ ) [3–7]. In the first one, the magnetic moments are parallel to the ring axis, whereas in the other two they lay parallel to the ring basis. The onion state is accessible from saturation and is characterised by the presence of two opposite head to head walls [8,9]. On the other hand, in the vortex configuration the magnetic moments circulate the ring axis. We remark that in the  $c = 1$  and  $c = 2$  configurations, in which the ring magnetisation is nonzero, the resulting demagnetising field leads to deviations in the direction of the magnetic moments close to the particle borders, [6,10,11] giving rise to edge domains. Thus, the magnetic behaviour of the ring turns out to be sensitive to shape fluctuations and surface roughness, resulting in complicated switching processes. However, in the vortex configuration most of the magnetic flux is confined within the particle, and its magnetic behaviour is almost insensitive to surface imperfections. In addition, the absence of the highly energetic core region in rings stabilises the vortex state, leading to simpler and reproducible switching processes. As a consequence, the determination of occurrence of the vortex state in nanorings has been regarded as a key issue for the production of new magnetic devices.

In this work we investigate the magnetic properties of stacks of two magnetic nanorings separated by a non-magnetic one, as depicted in Figure 1. The geometry of such structures is characterised by the heights  $H_1$  and  $H_2$  of the lower and upper magnetic

layers, respectively, the thickness  $d$  of the non-magnetic spacer, and the external  $R$  and internal  $a$  radii. We are interested in determining the range of values of these parameters for which the magnetic nanorings in such layered structure exhibit one of the three characteristic configurations. In other words, we investigate the relative stability of the magnetic configurations of the whole structure, which can be identified by two indices  $[c_1, c_2]$ , where  $c_1, c_2 = 1, 2$  or  $3$  denote the internal configurations of the lower and upper magnetic nanorings, respectively. In what follows, we evaluate and compare the total energy  $E_{\text{tot}}^{[c_1, c_2]}$  of the various configurations as a function of ring's geometric parameters.

**Figure 1** Three characteristic magnetic configurations of layered nanorings



## 2 Model

We adopt a simplified description of the system, in which the discrete distribution of magnetic moments is replaced with a continuous one characterised by the magnetisation  $\vec{M}(\vec{r})$ . We recall that  $E_{\text{tot}}^{[c_1, c_2]}$  is generally given by the sum of three terms corresponding to the magnetostatic ( $E_{\text{dip}}^{[c_1, c_2]}$ ), exchange ( $E_{\text{ex}}^{[c_1, c_2]}$ ), and anisotropy contributions. In the case of soft or polycrystalline magnetic materials, which we consider here, the latter can be safely neglected.

In any case, the dipolar term can be written as

$$E_{\text{dip}}^{[c_1, c_2]} = \frac{\mu_0}{2} \int_v \vec{M}(\vec{r}) \cdot \nabla U(\vec{r}) dv, \tag{1}$$

where  $U(\vec{r})$  is the magnetostatic potential associated with the total magnetisation  $\vec{M}(\vec{r})$  [12]. We remark that  $\vec{M}(\vec{r}) = \vec{M}_1(\vec{r}) + \vec{M}_2(\vec{r})$ , where  $\vec{M}_1(\vec{r})$  and  $\vec{M}_2(\vec{r})$  are the magnetisation densities of the lower and upper nanorings, respectively. Thus, the magnetostatic potential splits up into two components,  $U_1(\vec{r})$  and  $U_2(\vec{r})$ , associated with the magnetisation of each individual nanoring. It follows that  $E_{\text{dip}}^{[c_1, c_2]} = E_{\text{dip}}^{[c_1]}(1) + E_{\text{dip}}^{[c_2]}(2) + E_{\text{int}}^{[c_1, c_2]}$ , where

$$E_{\text{dip}}^{[c_i]}(i) = \frac{\mu_0}{2} \int_{v_i} \vec{M}_i(\vec{r}) \cdot \nabla U_i(\vec{r}) dv, \quad i = 1, 2 \tag{2}$$

is the dipolar contribution to the self-energy of the ferromagnetic nanoring  $i$ , and

$$E_{\text{int}}^{[c_1, c_2]} = \mu_0 \int_{V_1} \vec{M}_1(\vec{r}) \cdot \nabla U_2(\vec{r}) dV \quad (3)$$

is the dipolar interaction between the lower and upper ferromagnetic nanorings. On deriving equation (3) we have used the reciprocity theorem [12].

In the case of rings in the configurations under consideration, the magnetisation can be regarded as varying slowly on the scale of the lattice parameter. In addition, we focus on systems in which the thickness  $d$  of the intermediate layer is sufficiently large so as to make negligible the indirect exchange interaction between the two magnetic rings across the non-magnetic one. Such a condition is generally achieved when  $d \geq L_{\text{ex}}$ , with  $L_{\text{ex}}$  the exchange length of the magnetic material given by  $L_{\text{ex}} = \sqrt{2A/\mu_0 M_0^2}$ . Here,  $A$  is the stiffness constant of the magnetic material and  $M_0$  is the saturation magnetisation. In such a condition, the ferromagnetic rings interact just magnetostatically. Hence, the exchange contribution to the total energy reduces to  $E_{\text{ex}}^{[c_1, c_2]} = E_{\text{ex}}^{[c_1]}(1) + E_{\text{ex}}^{[c_2]}(2)$ , where

$$E_{\text{ex}}^{[c_i]}(i) = A \int_{V_i} [(\nabla m_{ix})^2 + (\nabla m_{iy})^2 + (\nabla m_{iz})^2] dV \quad (4)$$

is the exchange contribution to the self-energy of nanoring  $i$  [12]. Here,  $\vec{m}_i = (m_{ix}, m_{iy}, m_{iz}) = \vec{M}_i / M_0$  is the reduced magnetisation. In equation (4) a constant and configuration-independent term has been left out [12].

On the basis of the above results, the total energy can be written as  $E_{\text{tot}}^{[c_1, c_2]} = E_{\text{self}}^{[c_1]}(1) + E_{\text{self}}^{[c_2]}(2) + E_{\text{int}}^{[c_1, c_2]}$ , where  $E_{\text{self}}^{[c_i]}(i) = E_{\text{dip}}^{[c_i]}(i) + E_{\text{ex}}^{[c_i]}(i)$  is the self-energy of nanoring  $i$ .

We now proceed to the calculation of the various terms in the expression for the total energy. Results are given in units of  $\mu_0 M_0^2 L_{\text{ex}}^3$ , that is to say  $\tilde{E} = E / \mu_0 M_0^2 L_{\text{ex}}^3$ . We start by looking at the self-energy terms. For the out of plane magnetisation ( $c = 1$ )  $\vec{M}(\vec{r})$  can be approximated by  $M_0 \hat{z}$ , where  $\hat{z}$  is the unit vector parallel to the nanoring axis. It follows that the only non-zero contribution to the ring self-energy comes from the dipolar interaction. Thus, after some manipulations, we obtain

$$\tilde{E}_{\text{self}}^{[1]}(i) = \frac{\pi R^3}{L_{\text{ex}}^3} \int_0^\infty \frac{dq}{q^2} (1 - e^{-q\gamma_i}) [J_1(q) - \beta J_1(\beta q)]^2, \quad (5)$$

where  $\beta = a/R$ ,  $\gamma_i = H_i/R$ , and  $J_1(z)$  are Bessel functions.

Regarding the onion configuration ( $c = 2$ ), Landeros et al. [13] have shown that in such a case the magnetisation density is well described by a simple analytical model depending on a variational parameter  $n$ , whose value is determined so as to minimise the ring total energy. In their model, the reduced magnetisation is written as the sum of a radial ( $m_r = M_r/M_0$ ) and an azimuthal ( $m_\phi = M_\phi/M_0$ ) component satisfying the relations

$$m_r(\phi) = \begin{cases} f(\phi), & 0 < \phi < \pi/2 \\ -f(\pi - \phi), & \pi/2 < \phi < \pi \\ -f(\phi - \pi), & \pi < \phi < 3\pi/2 \\ f(2\pi - \phi), & 3\pi/2 < \phi < 2\pi, \end{cases} \quad (6)$$

and

$$m_\phi(\phi) = \begin{cases} -\sqrt{1-f^2(\phi)}, & 0 < \phi < \pi/2 \\ -\sqrt{1-f^2(\pi-\phi)}, & \pi/2 < \phi < \pi \\ \sqrt{1-f^2(\phi-\pi)}, & \pi < \phi < 3\pi/2 \\ \sqrt{1-f^2(2\pi-\phi)}, & 3\pi/2 < \phi < 2\pi, \end{cases} \quad (7)$$

with  $f$  given by

$$f(n, \phi) = \cos^n(\phi), \quad n \geq 1. \quad (8)$$

In such model,  $n = 1$  corresponds to an in-plane ferromagnetic configuration. Landeros et al. [13] have considered rings with different geometries and found that, whenever the onion configuration is the stable one,  $n$  turns out to be very close to 1. This is in agreement with the work of Beleggia et al. [14] in which the authors have argued that, as far as the magnetic phase diagram of nanorings is concerned, replacing the more correct quasi-uniform states with simple ideal uniform magnetisation should lead to errors in the position of the phase boundaries not exceeding 10%. In view of that, we set  $n = 1$  and find

$$\tilde{E}_{\text{self}}^{[2]}(i) = \frac{\pi R^3}{2L_{\text{ex}}^3} \int_0^\infty \frac{dq}{q^2} (e^{-q\gamma_i} + q\gamma_i - 1) [J_1(q) - \beta J_1(\beta q)]^2. \quad (9)$$

We remark that also in this case, the only non-zero contribution to the ring self-energy comes from the dipolar interaction.

Finally, in configuration  $c = 3$  the magnetisation can be approximated by  $M_0 \hat{\phi}$ . The contribution from the dipolar energy to the self-energy results equal to zero, and  $\tilde{E}_{\text{self}}^{[3]}(i) = -(\pi H_i \ln \beta) / L_{\text{ex}}$ .

With regard to the interaction between the magnetic nanorings, the only non-zero terms are those corresponding to the cases in which both rings are in the same configuration, either  $c = 1$  or  $c = 2$ . Due to the condition of perfect flux closure in the vortex state ( $c = 3$ ), one magnetic nanoring in such configuration does not interact with the other, independent of the magnetic configuration of the second one. In addition, one can easily show that the dipolar interaction between two nanorings in both the [1, 2] and [2, 1] configurations is zero. Thus, we are left with just two interaction terms, namely

$$\tilde{E}_{\text{int}}^{[1,1]} = -\frac{\pi R^3}{L_{\text{ex}}^3} \int_0^\infty \frac{dq}{q^2} [J_1(q) - \beta J_1(\beta q)]^2 e^{-q\frac{d}{R}} (1 - e^{-q\gamma_1}) (1 - e^{-q\gamma_2}) \quad (10)$$

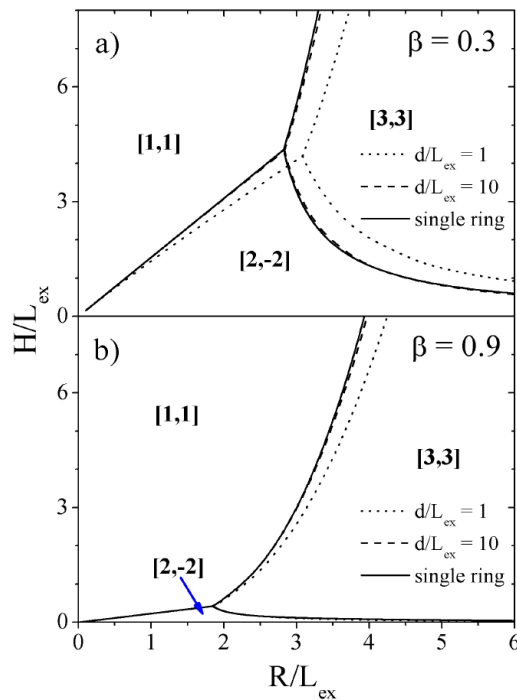
and  $\tilde{E}_{\text{int}}^{[2,-2]} = \tilde{E}_{\text{int}}^{[1,1]} / 2$ . The minus sign in the superscript of  $\tilde{E}_{\text{int}}^{[2,-2]}$  indicates that the alignment of the magnetisations of the lower and upper nanorings is antiparallel. We remark that the total energy of the [2, 2] configuration is always larger than that of the [2, -2] one.

### 3 Results and discussion

Having evaluated all contributions to the total energy in the cases of interest, we are in position to investigate the relative stability of the various configurations  $[c_1, c_2]$ .

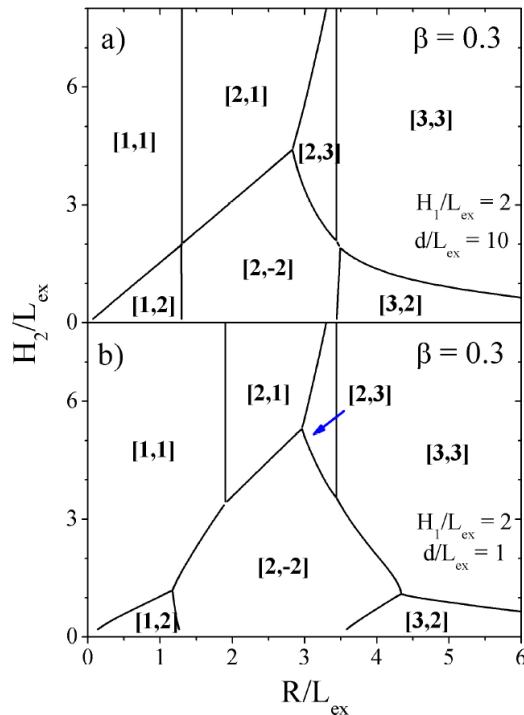
Let us consider firstly systems with  $H_1 = H_2 = H$ . Figure 2 shows the phase diagrams giving the regions in the  $RH$ -plane within which one of the  $[c_1, c_2]$ -configurations is of lowest energy. Results are presented for  $d/L_{\text{ex}} = 1$  (dotted lines),  $d/L_{\text{ex}} = 10$  (dashed lines), and  $d/L_{\text{ex}} \gg 1$  (solid lines), and for two values of  $\beta = a/R$ , namely  $\beta = 0.3$  (a) and  $\beta = 0.9$  (b). The case in which  $d/L_{\text{ex}} \gg 1$  corresponds to non-interacting rings. We notice that the diagrams show three regions, corresponding to configurations  $[1, 1]$ ,  $[2, -2]$  and  $[3, 3]$ , as in the case of a single nanoring [13]. It is worth mentioning that in our model the configurations  $[3, -3]$ , (vortex, anti-vortex) and  $[3, 3]$  (vortex, vortex) have equal energy, since in both cases the magnetic nanorings do not interact. In what follows we shall refer to both as  $[3, 3]$ -configurations. The boundary lines between the different configurations in Figure 2, as well as the position of the triple point in the diagram, change with  $\beta$ . An interesting result is the significant reduction in the size of the  $[2, -2]$  stability region as the ring walls are made thinner (as  $\beta$  approaches 1). In addition, the reduction in the value of  $d$  favours the stability of the  $[1, 1]$  and  $[2, -2]$  by increasing the interaction between the magnetic rings in the ferromagnetic configurations.

**Figure 2** Phase diagrams for layered nanoring with  $H_1 = H_2 = H$  giving the regions in the  $RH$ -plane within which one of the characteristic magnetic configurations is of lowest energy, for (a)  $\beta = a/R = 0.3$  and (b)  $\beta = a/R = 0.9$ . Results are presented for  $d/L_{\text{ex}} = 1$  (dotted lines),  $d/L_{\text{ex}} = 10$  (dashed lines), and  $d/L_{\text{ex}} \gg 1$  (solid lines) which corresponds to a single ring



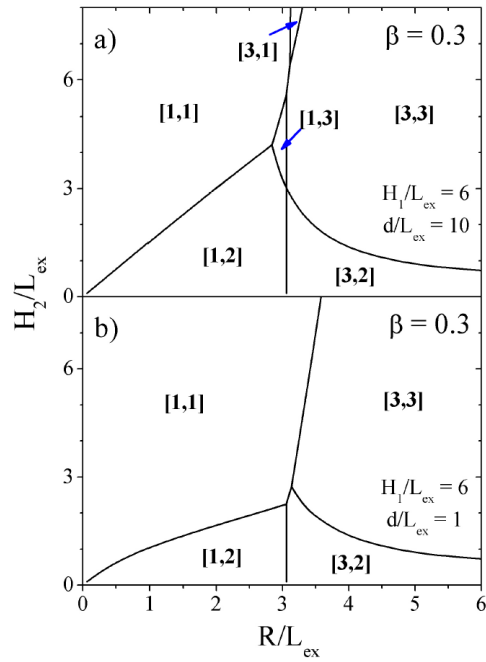
We now turn our attention to the case in which  $H_1$  and  $H_2$  are different. Figure 3 shows phase diagrams in the  $RH_2$ -plane for  $\beta=0.3$  and  $H_1/L_{\text{ex}}=2$ , and two different thicknesses of the non-magnetic ring, namely  $d/L_{\text{ex}}=10$  (a) and  $d/L_{\text{ex}}=1$  (b). We notice the occurrence of regions within which additional configurations such as [1,2], [2,3] and [3,1] are of lowest energy. It is therefore clear that asymmetric layered structures exhibit a more complex magnetic behaviour, which might be of interest from the point of view of applications.

**Figure 3** Phase diagrams for layered nanoring with  $H_1/L_{\text{ex}}=2$  and  $\beta=0.3$ , for (a)  $d/L_{\text{ex}}=10$  and (b)  $d/L_{\text{ex}}=1$

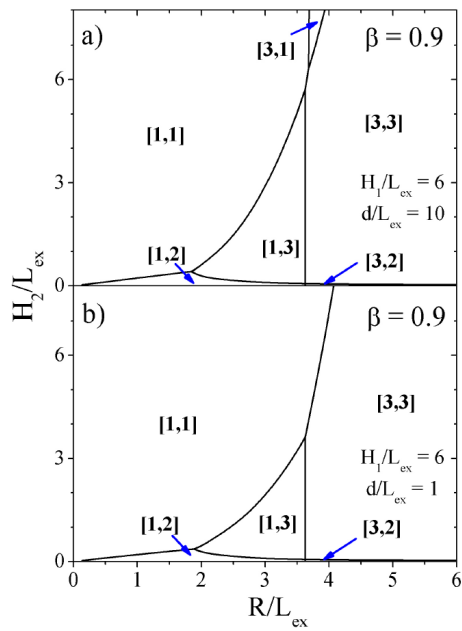


The phase diagrams in Figures 4 and 5 illustrate the possibility of controlling the magnetic behaviour of such layered structures by choosing appropriate values for the geometric parameters. In both cases,  $H_1/L_{\text{ex}}$  is set equal to 6 and results are presented for two different thicknesses of the spacer ring, namely  $d/L_{\text{ex}}=10$  (a) and  $d/L_{\text{ex}}=1$  (b). The data in Figure 4 are for thick walled rings ( $\beta=0.3$ ), whereas those in Figure 5 are for thin walled rings ( $\beta=0.9$ ). Besides the changes in size of the different stability regions as the geometric parameters are modified, a result which may be of relevance to the construction of devices based on such structures is the possibility of suppressing some specific phases or configurations by an appropriate choice of the geometric parameters. In both Figures 4 and 5 the lower ring is never found in configuration 2, contrary to what we have in Figures 2 and 3. It is also noticeable the significant reduction in size of the [1,2] stability region as the thickness of the ring wall is reduced, as shown in Figure 5.

**Figure 4** Phase diagrams for layered nanoring with  $H_1/L_{ex} = 6$  and  $\beta = 0.3$ , for (a)  $d/L_{ex} = 10$  and (b)  $d/L_{ex} = 1$



**Figure 5** Phase diagrams for layered nanoring with  $H_1/L_{ex} = 6$  and  $\beta = 0.9$ , for (a)  $d/L_{ex} = 10$  and (b)  $d/L_{ex} = 1$



## 4 Conclusions

In conclusion, we have investigated the relative stability of specific magnetic configurations of layered nanorings as a function of their geometry. Results are summarised in phase diagrams for different sets of geometrical parameters, which clearly indicate that magnetic behaviour of such structures can be tailored to meet specific requirements provided a judicious choice of such parameters is made. This includes not just the control over the number of magnetic configurations the system might exhibit, but also the possibility of suppressing specific configurations.

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