Persistent current magnification in a double quantum-ring system

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The electronic transport in a system of two quantum rings side-coupled to a quantum wire is studied via a single-band tunneling tight-binding Hamiltonian. We derived analytical expressions for the conductance, density of states, and the persistent current when the rings are threaded by magnetic fluxes. We found a clear manifestation of the presence of bound states in each one of those physical quantities when either the flux difference or the sum of the fluxes are zero or integer multiples of a quantum of flux. These bound states play an important role in the magnification of the persistent current in the rings. We also found that the persistent current keeps a large amplitude even for strong ring-wire coupling.

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I. INTRODUCTION

Electronic transport through quantum-ring structures has become the subject of active research during the last years. Interesting quantum interference phenomena have been predicted and measured in these mesoscopic systems in the presence of a magnetic flux, such as the Aharonov-Bohm oscillations in the conductance, persistent currents,[1–3] and Fano antiresonances.[4,5] Also, optical spectroscopy measurements have allowed a determination of the energy spectra of closed semiconducting rings.[6] Recently, Führer[7] reported magnetotransport experiments on closed rings showing the Aharonov-Bohm effect on the energy spectra.

Since the persistent currents in isolated mesoscopic rings were predicted[8] and posteriorly verified experimentally,[9] there have been many studies on the single- and coupled-ring systems. The persistent currents also have been reported in open systems. In these systems, the persistent current magnification effects have been predicted for several authors.[10–12] For instance Pareek and Jayannavar[10] reported persistent current magnification in two mesoscopic rings connected to an electron reservoir, Yi *et al.*[11] found giant persistent currents in open Aharonov-Bohm rings, and Bandopadhyay *et al.*[12] obtained persistent current magnification in multichannel mesoscopic rings.

A mesoscopic ring coupled to a reservoir was discussed theoretically a long time ago by Büttiker,[13] in which the reservoir acts as a source of electrons and an inelastic scatterer. Takai and Otha[14] considered the case where the magnetic flux and an electrostatic potential were applied simultaneously. The occurrence of persistent currents and the behavior of the electric conductance along a normal metal loop connected to two electron reservoirs was also discussed previously.[15] Recently, Wunsch and Chudnovsky[16] found the development of long-living states in a single ring coupled to a reservoir and discuss their effect in the persistent currents and the relation with the Dicke effect in optics.[17]

In a previous paper[5] we investigate the conductance and persistent current of a mesoscopic quantum ring attached to a perfect quantum wire in presence of a magnetic field. We show that the system develops an oscillating band with resonances (perfect transmission) and antiresonances (perfect reflection). In addition, an odd-even parity of the number of sites of the ring was found. With the advances in the fabrication techniques of nanostructures, it is natural to propose and study more complex mesoscopic ringlike structures, as a double quantum ring, and to develop a theoretical quantum-mechanical description for understanding such a different family of systems. In the present work we address the study of the transport properties of a system composed by two quantum rings side attached to a quantum wire. We found persistent current magnification in the rings due to the formation of bound states in the rings when the magnetic flux difference is an integer of the quantum of flux.

II. MODEL

The system under consideration is formed by two N-sites of quantum rings connected by tunnel coupling to a quantum wire waveguide of infinity length, as shown schematically in Fig. 1. The rings are threaded by a magnetic field flux. The full system is modeled by a single-band tight-binding Hamiltonian within a noninteracting picture, which can be written as

$$H = H_W + H_R + H_{WR}, \tag{1}$$

with

$$H_W = -v \sum_{\langle i \neq j \rangle} (c_i^\dagger c_j + c_i c_j^\dagger),$$

$$H_R = \sum_{n=1,\alpha=u,l}^{N} \varepsilon_n^\alpha d_{n,\alpha}^\dagger d_{n,\alpha}$$

$$H_{WR} = -V_0 \sum_{n=1,\alpha=u,l}^{N} (d_{n,\alpha}^\dagger c_0 + c_0^\dagger d_{n,\alpha}), \tag{2}$$

where $c_i^\dagger$ is the creation operator of an electron at site i of the wire and $d_{n,\alpha}^\dagger$ is the corresponding operator of an electron in the state n of the upper ($\alpha = u$) or lower ($\alpha = \ell$) ring. The wire site energy is assumed equal to zero and the hopping energies for wire and rings are taken to be equal to v, whereas V_0

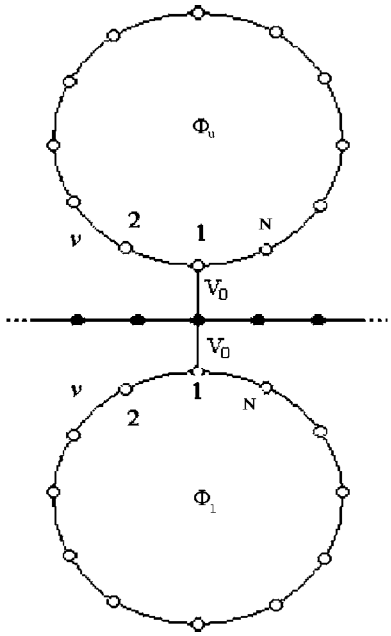

FIG. 1. Schematic view of the two quantum rings attached to quantum wire.

couples both systems. In the nearest-neighbor approximation the energy of the state n of the ring α is $\varepsilon_n^\alpha = 2v\cos[2\pi(n+\varphi_\alpha)/N]$, where $\varphi_\alpha = \Phi_\alpha/\Phi_0$, is the magnetic flux in unit of elemental quantum flux ($\Phi_0 = h/e$).

The linear conductance can be obtained from Landauer formula at zero temperature

$$\mathcal{G} = \frac{2e^2}{h} T(\omega = \varepsilon_F), \tag{3}$$

where $T(\omega)$ is the transmission probability, given by

$$T(\omega) = \frac{2\Gamma_L(\omega)\Gamma_R(\omega)}{\Gamma_L(\omega)+\Gamma_R(\omega)} \Im m[G_0^W], \tag{4}$$

where G_0^W is the Green's function of site 0 of the wire and $\Gamma_{L(R)}$ is the coupling of site 0 with the left (right) side of the wire.

By using a Dyson equation $G = g + gH_{WR}G$ we calculate the Green's function of site 0 of the quantum wire coupled to the rings, obtaining the following expression:

$$G_0^W = \frac{i}{2v\sin(k)} \frac{1}{1 - i\gamma(Q_1+Q_2)}, \tag{5}$$

where $\gamma = \pi V_0^2/2v\sin(ka)$ $[ka = \arccos(-\omega/2v)]$ and Q_α is given by the following equation:

$$Q_\alpha = \sum_{n=1}^{N} \frac{1}{\omega - \epsilon_n^\alpha} = \frac{1}{2v}\frac{\sin(Nka)}{\sin(ka)}[\cos(2\pi\varphi_\alpha) - \cos(Nka)]^{-1}. \tag{6}$$

By considering symmetric couplings $\Gamma_L = \Gamma_R = \Gamma(\omega) = 2v\sin(ka)$ and Eqs. (3)–(5), we evaluate the linear conductance,

$$\mathcal{G} = \frac{2e^2}{h}\Gamma(\omega)\Im m[G_0^W(\omega)]\big|_{\omega=\varepsilon_F} = \frac{2e^2}{h} \left.\frac{1}{1+\gamma^2(Q_1+Q_2)^2}\right|_{\omega=\varepsilon_F}. \tag{7}$$

The density of states (DOS) of the quantum rings can give us a better understanding of the transport properties of the system. To obtain it we calculate the spectral function S_n^α from the imaginary part of the diagonal elements of the Green's functions of the rings, G^α ($\alpha = u, l$); this is

$$G_n^\alpha = g_n^\alpha + \frac{i\gamma|g_n^\alpha|^2}{1 - i\gamma(Q_u+Q_l)}, \tag{8}$$

then

$$S_n^\alpha(\omega) = \frac{1}{\pi}\frac{\gamma}{(\omega-\varepsilon_n^\alpha)^2}\frac{1}{1+\gamma^2(Q_u+Q_l)^2}. \tag{9}$$

The density of states in each ring is obtained summing over all the states of the corresponding ring,

$$\rho_\alpha(\omega) = \sum_{n=1}^{N} S_n^\alpha(\omega) = \frac{\gamma}{\pi}\frac{(-\partial Q_\alpha/\partial\omega)}{1+\gamma^2(Q_u+Q_l)^2}. \tag{10}$$

In the presence of a magnetic flux threading the rings, a persistent current is generated through the circular system. The persistent current density j_α in the ring α, in the energy interval $d\omega$ around ω, is given by

$$j_\alpha(\omega) = \frac{e}{h\pi}\sum_{n=1}^{N}\left(-\frac{\partial\varepsilon_n^\alpha}{\partial\varphi_\alpha}\right)S_n^\alpha(\omega) = \frac{e\gamma}{h\pi}\frac{(-\partial Q_\alpha/\partial\varphi_\alpha)}{1+\gamma^2(Q_u+Q_l)^2}. \tag{11}$$

The total persistent current in each ring α is obtained, integrating the persistent current density over ω, we can write

$$I_\alpha = \int j_\alpha(\omega)f(\omega)d\omega, \tag{12}$$

where j_α is given by Eq. (11) and $f(\omega)$ is the Fermi-Dirac distribution. In order to study the behavior of the above physical quantities as a function of the relevant flux parameters, we adopt phases defined in terms of the Aharonov-Bohm phases φ_α: an added phase $\tilde{\varphi} = (\varphi_u + \varphi_l)$ and a difference phase $\delta\varphi = (\varphi_u - \varphi_l)$. Introducing the following definitions:

$$x \equiv \cos(\pi\tilde{\varphi})\cos(\pi\delta\varphi) - \cos(Nka),$$

$$y \equiv \sin(\pi\tilde{\varphi})\sin(\pi\delta\varphi),$$

$$\beta \equiv \frac{1}{2v}\frac{\sin(Nka)}{\sin(ka)}, \tag{13}$$

the conductance can be written as

$$\mathcal{G} = \frac{2e^2}{h} \left. \frac{[(x+y)(x-y)]^2}{[(x+y)(x-y)]^2 + (\gamma\beta)^2 x^2} \right|_{\omega=\varepsilon_F}. \quad (14)$$

From these equations we note that the conductance vanishes each time that $(x+y)=\cos(2\pi\varphi_l)-\cos(Nka)$ or $(x-y)=\cos(2\pi\varphi_u)-\cos(Nka)$ are zero. Therefore the conductance will present antiresonances for energies corresponding exactly to the energy spectrum of the isolated rings. On the other hand, the resonances in the conductance are obtained when β or x becomes null, except when y is equal to zero, which occurs when $\tilde{\varphi}$ or $\delta\varphi$ are zero or integer values.

Similarly the density of states (DOS) and persistent current density (PCD) can be written in terms of flux parameters as

$$\rho_\alpha(\omega) = \frac{N\gamma(x \pm y)^2}{\sin(ka)^2} \frac{[1 - \cos(Nka)\cos(\pi(\tilde{\varphi} \pm \delta\varphi)) - (2v\beta/N)(x \mp y)]}{(x^2-y^2)^2 + (2\gamma\beta)^2 x^2}, \quad (15)$$

$$j_\alpha(\omega) = -\frac{2e}{h} \frac{v \sin(\pi(\tilde{\varphi} \pm \delta\varphi))(x \pm y)^2}{(x^2-y^2)^2 + (2\gamma\beta)^2 x^2} \gamma\beta. \quad (16)$$

In the above expressions the upper (lower) sign corresponds to $\alpha=u(l)$. These analytical results permit us to analyze its behavior as a function of the different relevant parameters of the system. We can see that both functions oscillate with the energy and the magnetic flux parameters and that for each ring, DOS and PCD vanish at the eigenenergies corresponding to the other isolated ring, except when $y=0$. In particular, it can be observed that the PCD in both rings vanishes for the zeroes of β [Eq. (13)], i.e., $\omega=-2v\cos(\pi j/N)$ $(j=1,\ldots,N)$. Furthermore, the maxima amplitude of the DOS and PCD can be obtained from the minimum of the common denominator in Eqs. (15) and (16), i.e., $(x^2-y^2)^2+(2\gamma\beta)^2x^2$.

In what follows we present results for the conductance, DOS, and PCD of a double-ring system of $N=10$ sites, coupled through a wire. Energies are measured in units of the hoping parameter v. Note that all studied physical quantities depend on a completely equivalent form of the flux parameters $\tilde{\varphi}$ and $\delta\varphi$; therefore, it can be analyzed as a function of any of them and the wire-ring coupling energy γ. For a given set of parameters, the energy spectrum consists of a superposition of quasibound states reminiscent of the corresponding localized spectrum of the isolated rings.

Figure 2 displays the linear conductance versus the Fermi energy for a coupling energy $\gamma=0.5$ and parameters of magnetic flux given by $\tilde{\varphi}=0.6$ and $\delta\varphi=0.1$ (upper layer) and $\delta\varphi=0.2$ (lower layer). As expected from the analytical expression [Eq. (13)], the linear conductance presents a series of resonances and antiresonances as a function of the Fermi energy. The zeroes in the conductance represent exactly the superposition of the spectrum of each isolated ring $\omega=-2v\cos[2\pi(n+\varphi_{u(\ell)})/N]$. Note that when the difference between both fluxes decreases, the shape of the resonances changes drastically and the conductance becomes composed of broad and narrow peaks. When the magnetic fluxes coincide, the isolated-ring spectra are degenerated and the narrow peaks are completely suppressed from the conductance. We will analyze, in detail, this situation below.

In Fig. 3 we have plotted the density of states (upper plot) and the persistent current density (lower plot) as a function of energy ω, for $\gamma=0.5$ and flux parameters given by $\tilde{\varphi}=0.6$ and $\delta\varphi=0.1$. The solid and dotted lines indicate the upper and lower rings, respectively. We can see the expected structure of maxima and minima of DOS and PCD localized in the energy values defined by the spectrum of each isolated ring. The wire-ring coupling energy determines the mixing between the states of the two rings and in consequence the width of the peaks in the DOS and PCD.

An interesting situation appears when the energy spectrum of both rings becomes degenerated. This occurs when the magnetic fluxes threading the rings are equal or integer multiples of each other. This is $\tilde{\varphi}=M$ or $\delta\varphi=M$, with $M=0,\pm1,\pm2\ldots$. For this case $y\equiv\sin(\pi\tilde{\varphi})\sin(\pi\delta\varphi)=0$, and we obtain

$$\mathcal{G} = \frac{2e^2}{h} \frac{x^2}{x^2+4\gamma^2},$$

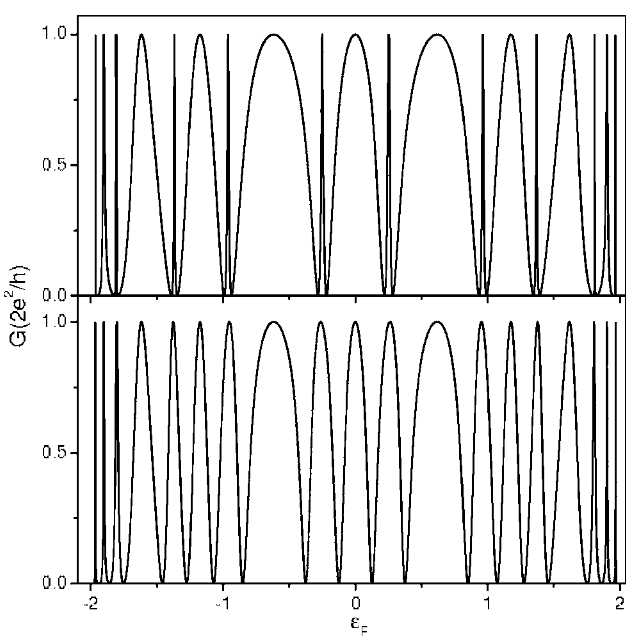

FIG. 2. Linear conductance as a function of Fermi energy measured in units of the hoping parameter v, for $\gamma=0.5v$, $\tilde{\varphi}=0.6$, and for a difference of fluxes between both rings of $\delta\varphi=0.1$ (top) and $\delta\varphi=0.2$ (bottom).

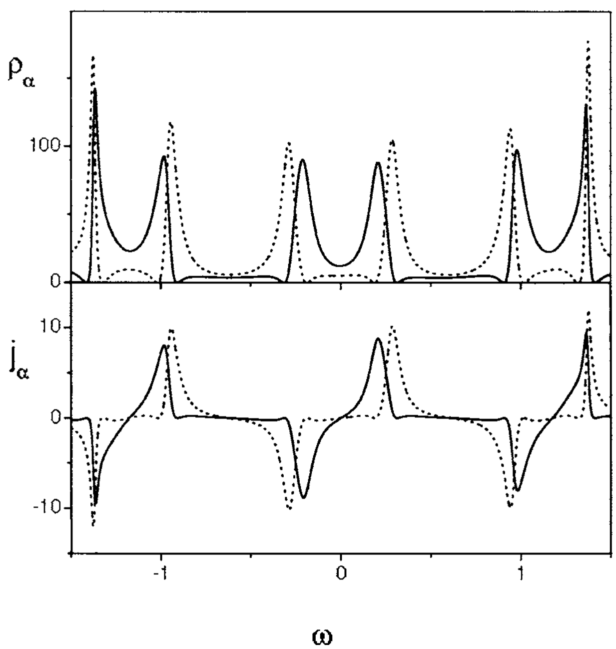

FIG. 3. Density of states (top) and persistent density current (in units of v), for $\tilde{\varphi}=0.6$, $\delta\varphi=0.1$, and $\gamma=0.5$. The solid and dotted lines indicate upper and lower rings, respectively.

$$\rho_\alpha(\omega)=\rho_s(\omega)+\sum_{n=1}^{N}\delta(\omega-\varepsilon_n),$$

$$j_\alpha(\omega)=j_s(\omega)+\frac{e}{h}\sum_{n=1}^{N}\left(-\frac{\partial\varepsilon_n}{\partial\tilde{\varphi}}\right)\delta(\omega-\varepsilon_n), \tag{17}$$

where ρ_s and j_s are given by

$$\rho_s(\omega)=\frac{N\gamma}{\sin(ka)^2}\frac{(1-\cos(Nka)\cos(\pi\tilde{\varphi})-(2v\beta/N)x)}{x^2+4\gamma^2},$$

$$j_s(\omega)=-\frac{e}{h}\frac{2\gamma\sin(\pi\tilde{\varphi})}{x^2+4\gamma^2}. \tag{18}$$

The analytical expression obtained for DOS clearly shows the presence of N bound states reminiscent of the spectrum of an isolated ring. It is clear that they do not contribute to the conductance. Bound states appear superposed to the corresponding spectrum of an effective single-ring coupling to the quantum wire with coupling energy equal to 2γ. Figure 4 illustrates the behavior of the DOS as a function of the energy for $\delta\varphi=0.0$. Here $\gamma=0.5$ and $\tilde{\varphi}=0.6$. The DOS shows the superposition of sharp and broad resonances centered at the same energies. It is important to note that the above expressions are valid only if $\gamma\neq 0$.

The presence of bound states induces divergences in the persistent current density for energies corresponding to the eigenenergies of the isolated rings. This affects the behavior of the total persistent current inside the rings, which is strongly enhanced for certain values of the parameters as we will see below. The total persistent current for the case of equals magnetic fluxes, i.e., $\delta\varphi=0.0$ can be written as

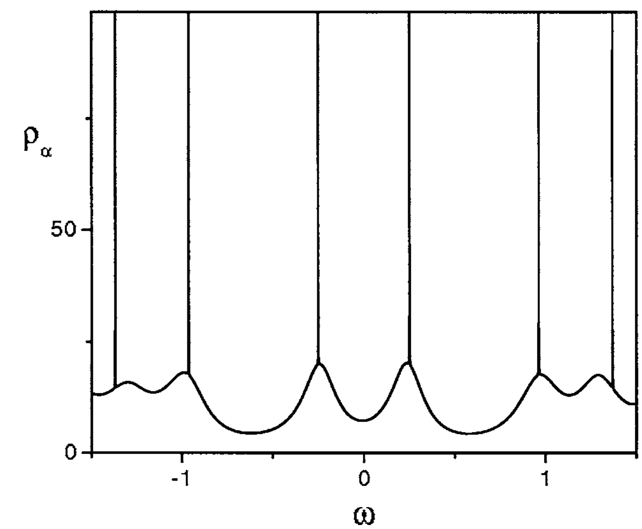

FIG. 4. Density of states as a function of energy (in units of v), for $\tilde{\varphi}=0.6$, $\delta\varphi=0.0$ and $\gamma=0.5$.

$$I_\alpha=\int j_s(\omega)f(\omega)d\omega+\frac{e}{h}\sum_{n=1}^{N}\left(-\frac{\partial\varepsilon_n}{\partial\tilde{\varphi}}\right)\Theta(\varepsilon_F-\varepsilon_n),$$

$$I_\alpha=I_s(\gamma,\tilde{\varphi})+I_b(\tilde{\varphi}), \tag{19}$$

where $j_s(\omega)$ is given by Eq. (18). Note that the persistent current in each ring contains two contributions, one I_s from a single ring with an effective coupling energy 2γ and another one I_b, independent of the coupling parameter. In order to evaluate the total persistent current we put the Fermi energy at the center of the band, i.e., $\varepsilon_F=0$ at zero temperature. In Fig. 5 we illustrate the total persistent current in units of the persistent current of an isolated ring I_0, for different values of γ. It can be observed that for small values of γ there is a notable enhancement of the total persistent current in each

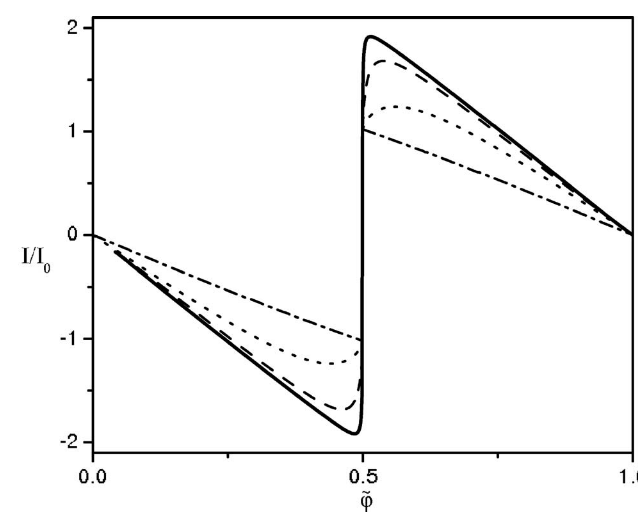

FIG. 5. Persistent current, in units of the persistent current of an isolated ring I_0, as a function of $\tilde{\varphi}$ for different values of ring-wire coupling, $\gamma=0.001$ (solid line), $\gamma=0.1$ (dashed line), $\gamma=1$ (dotted line), and $\gamma=100$ (dashed-dotted line).

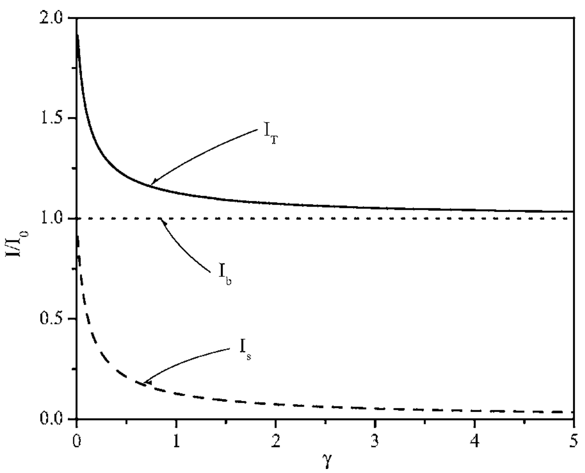

FIG. 6. Amplitude of the persistent current (solid line) and I_s (dashed line) and I_b (dotted line) as a function of γ measured in units of v.

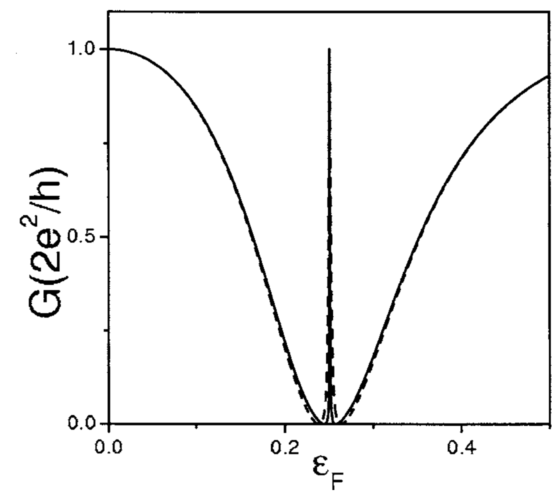

FIG. 7. Linear conductance as a function of ε_F (in units of v) for $\delta\varphi=0.01$ (solid line) and $\delta\varphi=1.99$ (dashed line).

ring. We observe that the amplitude of the persistent current is nearly twice the corresponding amplitude of the persistent current of an isolated ring I_0. This magnification effect is associated with the presence of bound states in the spectrum of the coupled-ring system.

Another interesting effect occurs for large values of γ. We observe that when γ increases, the amplitude of the persistent current tends to I_0; this is contrary to the case of a single ring connected to reservoirs for which the amplitude of the persistent current decays to zero for increasing γ. We show this effect in Fig. 6, which displays the amplitude of the total persistent current I_T and the amplitude from the contributions I_s and I_b as a function of coupling parameter. We see that for a sufficiently large γ, the amplitude of I_s tends to zero and the persistent current tends to I_0.

Now we examine the cases for $\delta\varphi$, taking values close to an integer. For these cases $y\approx 0$ and the conductance can be written approximately as a superposition of a broad Fano line shape and a narrow Breit-Wigner line shape around $x=0$ with widths $\Gamma_+=2\gamma$ and $\Gamma_-=y^2/(2\gamma\beta)$, respectively. This is

$$\mathcal{G}\approx\frac{2e^2}{h}\left(\frac{x^2}{x^2+\Gamma_+^2}+\frac{\Gamma_-^2}{x^2+\Gamma_-^2}\right). \qquad (20)$$

This expression clearly shows the superposition of short- and long-living states developed in the rings. Figure 7 displays one of the narrow resonances of conductance for $\tilde{\varphi}=0.6$, $\delta\varphi=0.01$ (solid line), and $\delta\varphi=1.99$ (dashed line).

Similarly, the density of states and persistent current density can be written near $y=0$ in terms of contributions coming from the two kind of states,

$$\rho_\alpha(\omega)\approx\frac{N\gamma}{\sin(ka)^2}\frac{[1-\cos(Nka)\cos(\pi(\tilde{\varphi}\pm\delta\varphi))-(2v\beta/N)(x\mp y)](x\pm 2y)x}{(x^2-y^2)^2+\Gamma_+^2x^2}+2vN\beta\frac{\Gamma_-}{x^2+\Gamma_-^2}, \qquad (21)$$

$$j_\alpha\approx-\frac{e\sin[\pi(\tilde{\varphi}\pm\delta\varphi)]}{h}\left(\frac{x(x\pm 2y)\Gamma_+}{(x^2-y^2)^2+\Gamma_+^2x^2}+\frac{\Gamma_-}{x^2+\Gamma_-^2}\right). \qquad (22)$$

Note that to obtain the result for isolated rings ($\gamma=0$) it is necessary to take the limit γ tends to zero maintaining $y\neq 0$ in the above equations. In this limit, Γ_- tends to infinity, the second terms in Eqs. (21) and (22) vanish, Γ_+ tends to zero, and the result for isolated rings is recovered. In Fig. 8 we show an example of the DOS [Fig. 8(a)] and the current density [Fig. 8(b)] as a function of the energy for nearly equal magnetic fluxes, $\delta\varphi=0.01$, threading the upper ring (solid line) or the lower ring (dashed line). The coupling energy parameter is $\gamma=0.001$. In Fig. 9 it shows the total persistent current as a function of the total of magnetic flux for different values of γ in the regime of nearly equal magnetic fluxes $\delta\varphi=0.001$.

The apparition of quasibound states in the spectrum of the system is a consequence of the mixing of the levels of both rings, which are coupled indirectly through the continuum of states in the wire. A similar effect was discussed recently in a system with a ring coupled to a reservoir by Wunsch *et al.*[13] They relate this kind of collective state to the Dicke effect in optics. The Dicke effect in optics takes place in the spontaneous emission of a pair of atoms radiating a photon with a wavelength much larger than the separation between them.[14] The luminescence spectrum is characterized by a

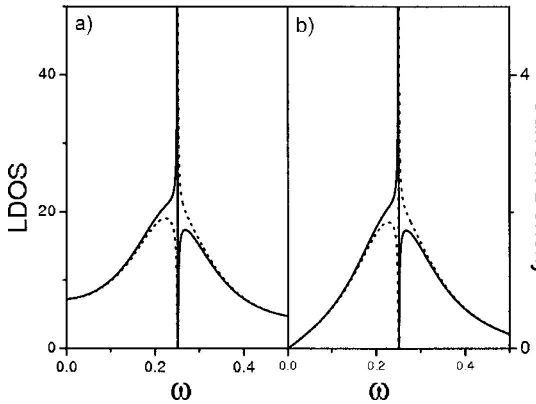

FIG. 8. (a) DOS and (b) current density in the upper ring (solid line) and lower ring (dashed line) for $\delta\varphi=0.01$ as a function of the energy (in units of v).

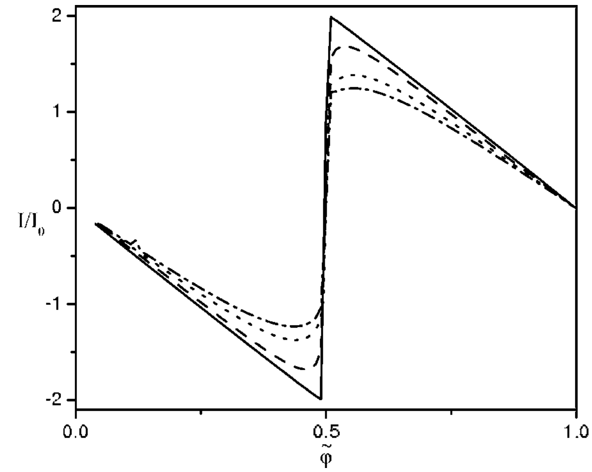

FIG. 9. Persistent current as a function of the magnetic flux for $\delta\varphi=0.001$ for different values of coupling parameter, $\gamma=0.0001$ (solid line), $\gamma=0.001$ (dashed line), $\gamma=0.5$ (dotted line), $\gamma=1$ (dashed-dotted line).

narrow and broad peak, associated with long- and short-lived states, respectively.

III. SUMMARY

We have investigated the persistent current, conductance, and density of states in a system of two side quantum rings attached to a quantum wire in the presence of magnetic fluxes threading the rings. In the regime of a nearly degenerated isolated ring spectrum, two kinds of collective states are developed in the system: states strongly coupled to the wire with an effective coupling twice the coupling of a single ring, and strongly localized states. We found that when either the flux difference or the sum of the fluxes are close to zero or close to integer multiples of a quantum of flux, both DOS and PCD show sharp and broad peaks around the energy corresponding to the eigenenergies of the isolated rings. In the limit when the isolated rings spectrum is completely degenerated, bound states are formed in the rings leading to a magnification of the persistent currents in the rings. We also found that the persistent current keeps a large amplitude even for strong ring-wire coupling. This is a purely quantum effect and is related to the formation of the collective states in the double quantum-ring system.

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