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On micromagnetic theory of thin cast amorphous microwires

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Abstract

The effect of the residual quenching stresses on the magnetization distribution in thin cast amorphous microwires with negative and positive magnetostrictions in zero magnetic field is studied. We find that different cast amorphous microwires with either negative or positive magnetostriction have total magnetization in zero magnetic field and neither present curling nor buckling modes. Also we estimate the resonance frequencies for the considered wires.

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Keywords: Cast amorphous glass-covered microwires; Magnetostriction; Magnetization

1. Introduction

Interest in cast amorphous glass-coated microwires has greatly increased in the last few years mainly due to their technological applications, in particular, as sensor elements in various devices. The Co-based compositions of cast amorphous glass-coated microwires with negative magnetostriction constant have good soft magnetic properties and significant giant magneto-impedance (GMI) characteristics. The simplest variant of the domain structure in cast amorphous glass-covered microwires with negative magnetostriction can be described in terms of a set of domains of elliptic disk forms perpendicular to wire axis as presented in Ref. [1].

On the other hand, the cast Fe-based microwires with a positive magnetostriction constant show rectangular hysteresis loop with a single and large Barkhausen jump between two stable magnetization states and exhibit the phenomenon of natural ferromagnetic resonance [2]. Moreover, the orientation of the magnetization vector for materials with positive magnetostriction is determined by the maximal component of the stress tensor; a particular domain structure in cast amorphous glass-covered microwire with positive magnetostriction was obtained in Ref. [3]. In addition, for glass-coated microwires with metallic core diameters of the order of 100 μm, it is known experimentally that positive magnetostriction wires have large remanence, while those with negative magnetostriction have almost vanishing remanence. However, when the diameter of the microwire metallic core is of the order of 1 μm, which is smaller than the position of a typical domain wall, we have a non-domain structure [4–6] and the magnetic properties change; in such case, the microwire can be represented as a simple monodomain and Brown's theory [4] becomes applicable.

In the present work, we analyzed the effect of the residual quenching stresses on the magnetization distribution in thin cast amorphous microwires with negative and positive magnetostrictions in zero magnetic field; in particular, we present interesting magnetic behavior for given thin cast amorphous glass-coated microwires with metallic nucleus radius $r_c \sim 1\,\mu$m. We generalize the cited Brown model for a cylinder; we find some exact

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solutions for a more real micromagnetism model, which to our knowledge has not been obtained by others. Besides, as application of the model, we estimate novel values of resonance frequencies. The paper is arranged in the following way: In Section 2 the theoretical model is developed. In Section 3 the resonance frequencies are estimated. Finally, the conclusions are presented in Section 4.

2. Theoretical model

In order to determine the remanent magnetization of the considered cast wires, and, therefore, estimate their resonance frequencies, we need to know the angle between the microwire axis and the vector magnetization, $\theta(\rho)$. By considering a model for stress formation in these wires, the relationships obtained from it will allow us to solve the generalized Brown equation:

$$\theta''(\rho)+\rho^{-1}\theta'(\rho)+\frac{1}{2}\{\eta/f(\rho)-\rho^{-2}\}\sin(2\theta(\rho))=0, \tag{1}$$

where ρ is the relative radial cylinder coordinate defined as $\rho=r/r_c$, under the restriction $\rho_0<\rho<1$, ρ_0 being the relative radius when the magnetization is homogeneous and parallel to the microwire axis, $\theta=0$, so it is determined to be $\rho_0^2=A/Kr_c^2$, where A is the exchange constant and K is the anisotropy energy of microwire. Besides, the parameter η has the value $\eta=1$ when the magnetostriction is negative and $\eta=-1$ when the magnetostriction is positive [6]. In addition, the angle θ has the restriction $0<\theta(\rho)<\pi/2$. The function $f(\rho)$ is determined by the stress distribution in cast wires; according to Ref. [7] it is

$$f(\rho)^{-1}=\rho_0^{-2}\{|\sigma_m-\sigma_z|/\sigma_z\}, \tag{2}$$

where, in cylindrical coordinates, σ_m corresponds to σ_ρ when the magnetostriction is negative and to σ_ϕ when the magnetostriction is positive.

We use the model for the stress formation in cast amorphous glass-covered microwires in which the electrochemical interaction is produced so that the outer surface of the strand adheres strongly to the glass shell and so, in the inner part of the core, there may be thermoplastic relaxation [1]; this model gives

$$\sigma_\rho=\sigma_0+P\left(1-(\rho_b/\rho)^2\right), \tag{3a}$$

$$\sigma_\phi=\sigma_0+P\left(1+(\rho_b/\rho)^2\right), \tag{3b}$$

$$\sigma_z=v(\sigma_\rho+\sigma_\phi), \tag{3c}$$

where σ_0 and P are parameters determined in Ref. [3], v is Poisson's coefficient (in elastic–plastic displacements case $v\sim(0.3, 0.5)$) and $\rho_b=b/r_c$, b being the ratio of the internal part of the metallic core which preserve liquid status. Since, for the considered wires, it was found $P\gg\sigma_0$, it is possible to approximate σ_z by σ_0+P and also ρ_0 by ρ_b. Hence for this model, an appropriated significant behavior of $f(\rho)^{-1}$ is

$$f(\rho)^{-1}\approx\rho_0^{-2}(\rho_b/\rho)^2. \tag{4}$$

Note that, for $\eta/f(\rho)=0$, Eq. (1) recovers the case of Brown equation for cylinder without anisotropy [4].

Once $\theta(\rho)$ is obtained, the remanent magnetization can be calculated by

$$M=2M_0\int_{\rho_{\rm inf}}^{\rho_{\rm sup}} \mathrm{d}\rho\,\rho\,\cos[\theta(\rho)], \tag{5}$$

where the upper and lower limits depend on the particular situation to be considered.

Let us first analyze the case when the magnetostriction is negative. By using the approximation $f(\rho)\approx\rho^2$ of Eq. (4), Eq. (1) can be simplified to [8]

$$\theta''(\rho)+\rho^{-1}\theta'(\rho)=0, \tag{6}$$

whose exact solution, with the conditions $\theta(\rho_0)=0$ and $\theta(1)=\pi/2$, can be written as

$$\theta(\rho)=c\,\ln(\rho/\rho_0), \tag{7}$$

where $c=\pi/(2\ln(1/\rho_0))$. Therefore, for the cast amorphous microwire with negative magnetostriction, the remanent magnetization, with $\rho_{\rm inf}=\rho_0$ and $\rho_{\rm sup}=1$, is approximately

$$M=2M_0\left(c-2\rho_0^2\right)(4+c^2)^{-1}. \tag{8}$$

Fig. 1 shows the remanent magnetization as a function of ρ_0; the inset shows θ as function of ρ for $\rho_0=0.1$. Note that M/M_0 increases slowly when ρ_0 tends to the upper limit $\rho_0=0.1$, and for this value we find that $M/M_0\approx0.29$; moreover, we notice that θ is a smooth function of ρ.

On the other hand, when the magnetostriction is positive, Eq. (1) gets the form

$$\theta''(\rho)+\rho^{-1}\theta'(\rho)-\frac{1}{2}(a/\rho)^2\sin(2\theta(\rho))=0, \tag{9}$$

where $a^2=(\rho_b/\rho_0)^2+1$; it has an exact solution (see Appendix A):

$$\tan\left(\theta/2\right)=\rho^a. \tag{10}$$

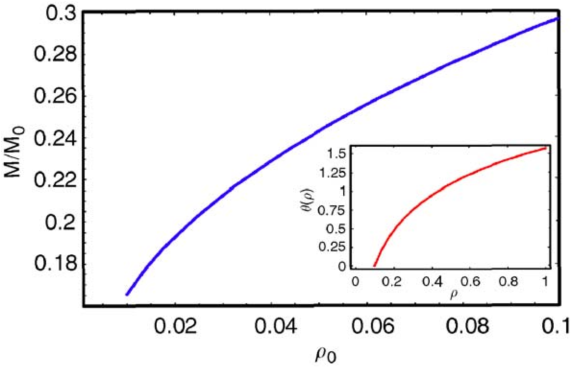

Fig. 1. Remanent magnetization as function of ρ_0. The inset shows the θ as function of ρ at $\rho_0=0.1$.

We remark that Eq. (10) satisfies exactly Eq. (9) at $\rho = 1$ since $\mathrm{Arctan}(1) = \pi/4$; and for very small ρ_0 it is the almost exact solution for our boundary value problem; so the magnetization can be expressed by

$$M = 2M_0 \int_{\rho_0}^{1} \mathrm{d}y\, y(1 - y^{2a})(y^{2a} + 1)^{-1}. \tag{11}$$

In explicit form, Eq. (11) can be cast as

$$\begin{aligned} M = M_0 a^{-1}\big(a - a\rho_0^2\big(1 - {}_2F_1\big[a^{-1}, 1, 1 + a^{-1}, -\rho_0^{2a}\big]\big) \\ + \Gamma[0, (2a)^{-1}] - \Gamma[0, (1 + a)/2a]\big), \end{aligned} \tag{12}$$

where ${}_2F_1[\alpha, \beta, \gamma, z]$ is a Hypergeometric function and $\Gamma[\alpha, \beta]$ is the Polygamma function. Note that, for $\rho_0 \ll 1$, Eq. (12) can be approximated by

$$M = M_0 a^{-1}(a + \Gamma[0, (2a)^{-1}] - \Gamma[0, (1 + a)/2a]). \tag{13}$$

Fig. 2 shows the absolute value of remanent magnetization as function of a for the case of positive magnetostriction for $\rho_0 \ll 1$. The inset shows θ as function of ρ at $\rho_0 = 0.1$ for different values of a. Note that as a increases M/M_0 tends to one and for $a = 2$ we have $M/M_0 \approx 0.57$; however, when the numerical value of ρ_0 is increased, the magnetization decreases. Besides, we remark that θ is strongly dependent on a, as can be seen in the inset of Fig. 2.

Let us comment a little about our results: the present model is an extension of Brown's theory; therefore, they have common applicability. On the other hand, in Brown's model there are two characteristic sizes [4]: the classical size of domain wall $\Delta = \sqrt{A/K} \sim 10 - 0.1\,\mu\mathrm{m}$ and the limit of single domain radius $\alpha = 1.84M\sqrt{A/2\pi} \sim 0.1 - 0.01\,\mu\mathrm{m}$, where α is obtained from the linear approximation of Brown's equation. Our model is applicable in mathematical sense up to the first size and in the physical sense an interval between the first and second size. Also in absence anisotropy, $a^2 = 1$, the present Eq. (9) and Brown's equation give exact result $\tan(\theta/2) = \rho$, which does not depend on the model of a microwire. On the other hand, for very large anisotropy, $a > 1$, our model as well as Brown's model give nucleation, in our theory it is a soliton.

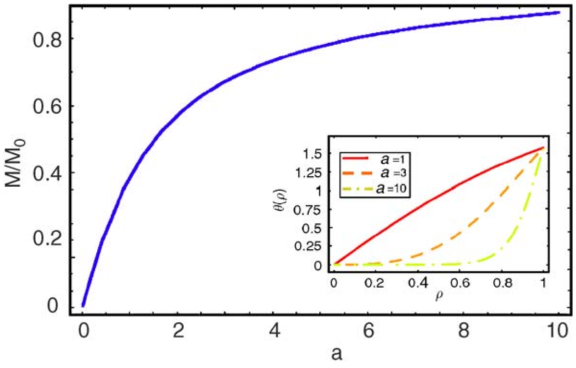

Fig. 2. Absolute value of remanent magnetization as function of a at $\rho_0 = 0.1$. The inset shows the θ as function of ρ at $\rho_0 = 0.1$ for different values of a.

These limiting show that our model confirms and supplements Brown's model.

3. Resonance frequency

As an application of our theoretical model, let us study resonance modes in magnetic microwires. This phenomenon has importance in the experimental characterization of the wires and it has a close relationship with the GMI [9] and the natural ferromagnetic resonance [2]. Since the problem of ferromagnetic resonance in a non-uniform magnetized cylinder has not been precisely solved, one resorts to estimations, which allow finding ranges where it is necessary to look for resonant frequencies. In particular, we calculate the reduced resonance frequency Ω [10]:

$$\Omega/2\pi = \gamma(H + \Delta N M_{\mathrm{eff}}), \tag{14}$$

where H is the magnitude of the applied magnetic field, γ is a frequency density, ΔN is the demagnetization factor and M_{eff} is the effective magnetization; for our case we use $\gamma = 2.8\,[\mathrm{MHz/Oe}]$ and $\Delta N \approx 2\pi$. When the applied magnetic field is null, Eq. (14) can be expressed as

$$\Omega \approx (2\pi)^2 \gamma M. \tag{15}$$

So, for $\rho_0 = 0.1$, the corresponding values of the reduced resonance frequency are 2 GHz when the cast amorphous microwire has a negative magnetostriction and 7 GHz when it has a positive magnetostriction, approximately; furthermore, the ranges of the reduced resonance frequency are 1–2 and 7–17 GHz, respectively. In work [11] natural ferromagnetic resonance in the case of microwires with zero magnetostriction in diapason 1–2 GHz was observed, which can be taken as an indication that the better the estimation given in Eq. (15) the larger the a^2.

4. Conclusion

The submitted model is applicable for a microwire, which does not present the domain structure. Both types of magnetostriction in cast amorphous microwires neither have curling nor buckling modes. We found that the considered cast amorphous microwires have finite magnetization in zero magnetic field. Furthermore, for Co-based and Fe-based cast amorphous microwires the resonance frequencies are approximately 2 and 7 GHz, respectively. As a final point, we suggest that these types of micorwires could adequately be used as radio absorbing materials [12], HF sensors and memory elements.

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Appendix A

The replacement of the structure

$$\tan(\theta/2) = \rho^{\beta} \tag{A.1}$$

in Eq. (9) on the text

$$\theta''(\rho) + \rho^{-1}\theta'(\rho) - \frac{1}{2}(a/\rho)^2 \sin(2\theta(\rho)) = 0 \tag{A.2}$$

gives

$$(\beta - a)(\beta + a) = 0. \tag{A.3}$$

Therefore, Eq. (A.2) has two exact solutions:

$$\tan\big(\theta/2\big) = \rho^{\pm a}. \tag{A.4}$$

The one with the minus sign provides the exact solution for the boundaries $\theta(0) = \pi$ and $\theta(1) = \pi/2$, while the plus sign provides for the boundaries $\theta(0) = 0$ and $\theta(1) = \pi/2$. Note that if we take a very small ρ_0 we reproduce the second boundary value problem, which corresponds to our physical case; as a numerical example of the previous, if we take $\rho_0 = 0.1$ we find $\theta(\rho_0) \approx 0.02$ and $\theta(\rho_0) \approx 2 \times 10^{-10}$ for $a = 2$ and $a = 10$, respectively. Since we have an analytical exact solution for the nonlinear boundary value problem we neither need to calculate the eigenvalues numerically nor resort to a linear approximation for calculating important modes of eigenstates.

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