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Dynamical behavior of two interacting magnetic nanoparticles

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Abstract

In the present work, we study the deterministic spin dynamic of a two interacting magnetic particles, with both dipolar and exchange interactions in the presence of an applied magnetic field. Due to the strength ratio of interactions, two time scales appear; a longer one associated with the exchange interaction and a shorter time scale associated with the dipolar interaction. We find that the magnitude of the total system magnetization is not constant; furthermore it is a fluctuating time-dependent function.

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Keywords: Spin dynamics; Dipolar interaction; Exchange energy

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1. Introduction

The world of nanometric scale increasingly gains access due to the remarkable development of experimental techniques. The technological applications of nanostructures can be found in many different areas, such as biomedicine or instrumentation. In material science, one significant application of magnetic particles and clusters is in the area of recording media [1]; and so the magnetization reversal is one of the fundamental features in data storage. The standard approaches to study the dynamics of the magnetization reversal are the Landau–Lifshitz (LL) [2] or the Landau–Lifshitz–Gilbert (LLG) equation [3]. Recent works of the stochastic LL or LLG equations, where the thermal fluctuation takes the role of the noise, are high-quality exposed in the Refs. [4,5], for a single particle and multiple particles, respectively.

Nonlinear time-dependent problems in magnetism have already been studied in many cases, in Ref. [6] there is an excellent description. A particular case of the LL model, the chaotical behavior of the magnetic moment for one anisotropic magnetic particle in a parametric magnetic field is reported in Ref. [7]; moreover, a good treatment of the nonlinear aspects of one magnetic particle under circularly polarized field is reported in Ref. [8] and a perturbation technique for the LLG equation is developed in the Ref. [9] for a magnetic particle under circularly polarized field. From a theoretical point of view, the analytical solutions of the LL or LLG equations are difficult to find due to the nonlinearity of these equations, and only in few cases they have been obtained, as for example in the problem of one magnetic particle with uniaxial anisotropy in the presence of an applied magnetic field when the easy axis is parallel to the external field [4]; analytical solutions for different orientations of the easy axis respect to the magnetic field vector and in absence of damping can be found in Refs. [10,11].

The dynamic of interacting magnetic particles considered as monodomains is important to model devices for spintronic [12]. The understanding of the magnetic properties, such as hysteresis loops, magnetization reversion, ZFC and FC behavior are fundamental to model, for example, magnetic susceptibility or magnetoresistance. Therefore a detailed study of a simple interacting magnetic systems is really important and in order. In the present work, we

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analyze in detail the deterministic behavior of a two interacting magnetic particles, with dipolar and exchange interaction, in the presence of an applied magnetic field; we consider different regimens of the parameters for understanding corresponding magnetic behaviors. It has been remarked [10] that, when a magnetic system subjected to short field pulses and its dynamics is fast then the dissipative effects can be neglected; hence for short time interval we have here applied this feature. The paper is arranged in the following way: In the Section 2 the theoretical model is shown. In the Section 3 the numerical results are discussed. Finally, the conclusions are presented in Section 4.

2. Theoretical model

Let us consider a system of N magnetic particles and assume that each particle can be represented by a magnetic monodomain. The temporal evolution of this system can be modeled by the LL equation. When effects of dissipation are negligible, the LL equations can be written as

$$\mathbf{m}_i(t) = \gamma \mathbf{m}_i(t) \times \nabla_{\mathbf{m}_i}\mathscr{H}, \tag{1}$$

where $\mathbf{m}_i$ is the individual magnetic moment with $i = (1, \ldots, N)$, $\dot{q}$ represent the time-dependent derivative of q, γ is an effective gyromagnetic ratio, $\nabla_{\mathbf{m}_i}$ is the gradient operator respect to $\mathbf{m}_i$ and H represents the appropriate Hamiltonian for the system. In general, the effective magnetic field is defined by $\mathbf{H}_{\text{eff}} = -\nabla_{\mathbf{m}_i}\mathscr{H}$. We remark that this form of writing the LL equations has an intrinsic relationship with the Nambu equation governing the dynamics for a triplet of canonical variables with two motion constants [13]; in the case of a single magnetic moment, the triplet of canonical variables is given by $\mathbf{m}$ and the two motion constants are the Hamiltonian and the normalization stationary condition.

In the present work, we analyze two interacting magnetic particles including dipolar and exchange interactions in the presence of an external field; therefore, the Hamiltonian has the following form:

$$\mathscr{H} = -\sum_{i=1}^{2} \mathbf{H}\cdot\mathbf{m}_i - (J - d^{-3})\mathbf{m}_1\mathbf{m}_2 - 3d^{-3}(\mathbf{m}_1\cdot\hat{\mathbf{n}})(\mathbf{m}_2\cdot\hat{\mathbf{n}}), \tag{2}$$

where J is the exchange constant, d is the fixed distance between the two magnetic moment and $\mathbf{n}$ is a unitary vector along the direction joining the two particles. Therefore, the two LL equations are:

$$\mathbf{m}_1 = -\gamma\mathbf{m}_1 \times (\mathbf{H} + (J - d^{-3})\mathbf{m}_2 + 3d^{-3}(\mathbf{m}_2\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}), \tag{3a}$$

$$\mathbf{m}_2 = -\gamma\mathbf{m}_2 \times (\mathbf{H} + (J - d^{-3})\mathbf{m}_1 + 3d^{-3}(\mathbf{m}_1\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}). \tag{3b}$$

We note that these equations have nonlinear couplings due to the interaction terms. In addition, this system presents two scales in the magnitude of the magnetic interactions, because they are of different nature; consequently, two time scales appear. Furthermore, when the distance between the magnetic moment increases, the dominant terms in Eqs. (3) are due to the exchange and the external applied field, in that case the total magnetization precesses around the applied field direction and the individual magnetizations precesses around another direction which is defined by the sum of the external field and the exchange coupling constant times the total magnetization.

On the other hand, Eqs. (3) have two constraints: the individual magnetization magnitudes are constant. These constraints have an important consequence in the dynamical behavior: the global dynamics of each magnetic moment is reduced to a spherical section. So by using spherical coordinates $\{r, \theta, \phi\}$, each magnetic moment can be expressed by $\mathbf{m}_i = m^i\hat{\mathbf{r}}$, hence Eqs. (3) are reduced to four differential equations, and they can be written as

$$\dot{\theta}^i = \gamma(H_\phi - 3d^{-3}m^k n_\phi n_r), \tag{4a}$$

$$\dot{\phi}^i \sin\theta^i = -\gamma(3d^{-3}m^k n_\theta n_r + H_\theta), \tag{4b}$$

where $i, k = 1, 2$ such that $k \neq i$. In general, the right-hand side of Eqs. (4) are complex functions of the angles. We observe that, in this representation the dynamics of each individual magnetic moment is almost uncoupled, since the only term that couples is a constant, which is the magnitude of the magnetization of the other magnetic moment.

As a final comment of this section, we remark that due to the dipolar interaction, the magnitude of the total magnetization is not stationary. In order to elucidate this property we can write $\mathbf{M} = \mathbf{m}_1 + \mathbf{m}_2$, so the dynamic of $\mathbf{M}$ obeys the following equation:

$$\dot{\mathbf{M}} = -\gamma\mathbf{M} \times (\mathbf{H} + 6d^{-3}(\mathbf{M}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}}) + \mathbf{f}\{\mathbf{m}_1, \mathbf{m}_2\}. \tag{5}$$

Eq. (5) describes the behavior of an equivalent single magnetic moment, with uniaxial anisotropy, in the presence of the applied magnetic field and of an extra field, $\mathbf{f}$, which is a function of $\mathbf{m}_1$ and $\mathbf{m}_2$. This extra field can be interpreted as a time-dependent fluctuating field. Moreover, from Eq. (5) it is clear that the magnitude of the total magnetization is not conserved, it is fluctuating in time.

3. Numerical solutions

In order to integrate numerically Eqs. (3), we express them in a dimensionless form; for this purpose we introduce the following new variables $\mathbf{e} = \mathbf{m}/m, \mathbf{h}_{\text{eff}} = \mathbf{H}_{\text{eff}}/H_0, \tau = t/(\gamma H_0)$, where H_0 is the magnitude of a reference magnetic field. The initial conditions for the magnetic moments are chosen a little far from the equilibrium points, so $\mathbf{e}_1(0) = (0.1, -0.1, 0.989949)$ and $\mathbf{e}_2(0) = (-0.1, 0.1, 0.989949)$ for all the situation studied; besides the fixed parameters are chosen to be $\mathbf{h} = 5\hat{\mathbf{z}}$ and

$\mathbf{n} = (0.382, 0, 0.924)$. We will assume that, the two magnetic particles are identical. An order four Runge–Kutta numerical method was used to solve the equations.

Among many other cases, firstly we show two particular cases, one in which the dipolar term is more important than the exchange term (DE) and a second one in which the exchange term is more important than the dipolar term (EE). The numerical values of the parameters in the first case are $Jm/H_0 = 3$ and $m/(d^3H_0) = 10^3$, in the second case they are $Jm/H_0 = 10^2$ and $m/(d^3H_0) = 0$.

The Fig. 1 shows the time evolution of the tree components of each individual magnetic moment where (a) is for DE and (b) is for EE. We note that, for the shown time scale, the magnetic moment components are quasi-periodic, and their initial dephasing is preserved. We remark that, the time scales for this to cases are clear in Fig. 1: in order to display adequately the time evolution we needed to use for EE case a time interval 10 times that for the other, so that time scale for the dynamics of the EE case is slower than the time scale for the DE case.

The Fig. 2 shows the parametric trajectory of $\mathbf{e}_1$ in the phase-space, where (a) is for DE and (b) is for EE. Note that the shape of the surface has a hole centered on an axis which is rotated with respect to the z-axis; this hole is a manifestation of the interaction. We note that, in the EE case the dynamics evolution is almost confined on a ring of a spherical surface in the three-dimensional phase space.

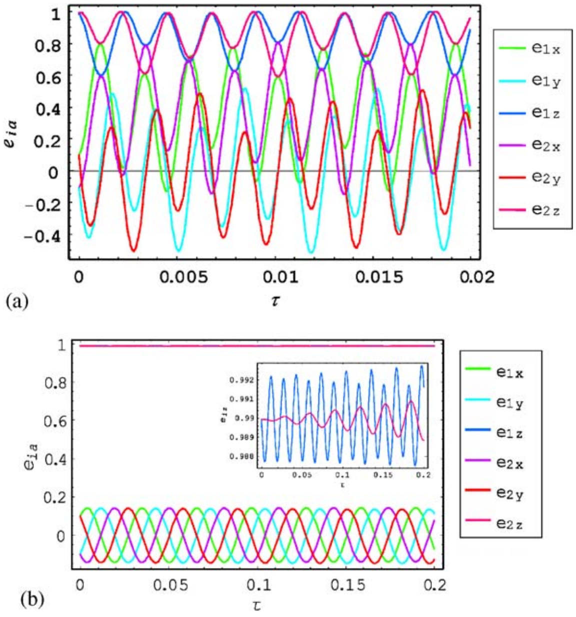

Fig. 1. (a) The magnetization components as function of time for $Jm/H_0 = 3$ and $m/(d^3H_0) = 10^3$. (b) The magnetization components as function of time for $Jm/H_0 = 10^2$ and $m/(d^3H_0) = 3$. The inset shows a zoom of the z-component of each magnetic moment as function of time.

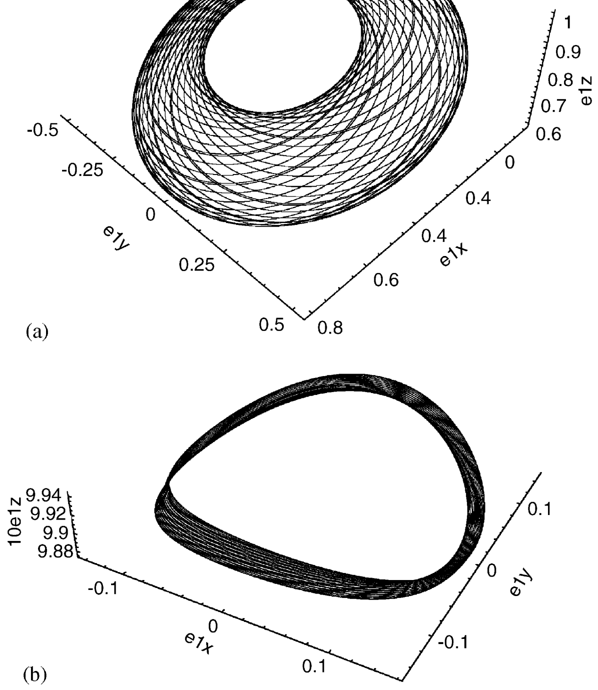

Fig. 2. (a) 3D phase diagram of components of $\mathbf{e}_1$ for $Jm/H_0 = 3$ and $m/(d^3H_0) = 10^3$. (b) 3D phase diagram of components of $\mathbf{e}_1$ for $Jm/H_0 = 10^2$ and $m/(d^3H_0) = 3$.

Now let us next analyze the dynamical behavior of the modulus of the total magnetization. The Fig. 3 shows the total magnetization as function of time, for a fixed value of the dipolar parameter, $m/(d^3H_0) = 10^2$, and different values of the exchange parameter in the range $Jm/H_0 = (0, 20)$. Notice that, in the set of Figs. 3 the magnitude of the total magnetization function is not constant on time, which is a consequence of the dipolar term. Moreover, the dependence on the exchange interaction is included in the extra field in concordance with Eq. (5); then for a constant dipolar term, the influence of the exchange interaction can be appreciated examining the envelope function of the total magnetization in Fig. 3(a)–(e). The Fig. 3(a) correspond to the null exchange interaction for which the envelope function is almost a ribbon, but then when the exchange parameter, in the considered range, is increased the envelope function show harmonic forms with increasing period, as exhibited in Fig. 3(b)–(e). The Fig. 3(f) shows a reduced time interval behavior corresponding to Fig. 3(a)–(e), where the dashed line is for (e); we note that, when the exchange parameter increased the periodicity slowly increase and the amplitude difference between the upper and lower values in each period increases too. Therefore, depending on the relative strength of these interactions we find two different time scales for both the dynamical behavior in each magnetic particle and for the modulus of the total magnetization and its envelope function.

In summary, when the exchange energy is the significant term in the Hamiltonian the orbits in the phase space of the individual particles are confined in the plane perpendicular to the external field; in the direction of the applied field their variations are almost negligible. However, when the dipolar energy is the important term, the phase-space

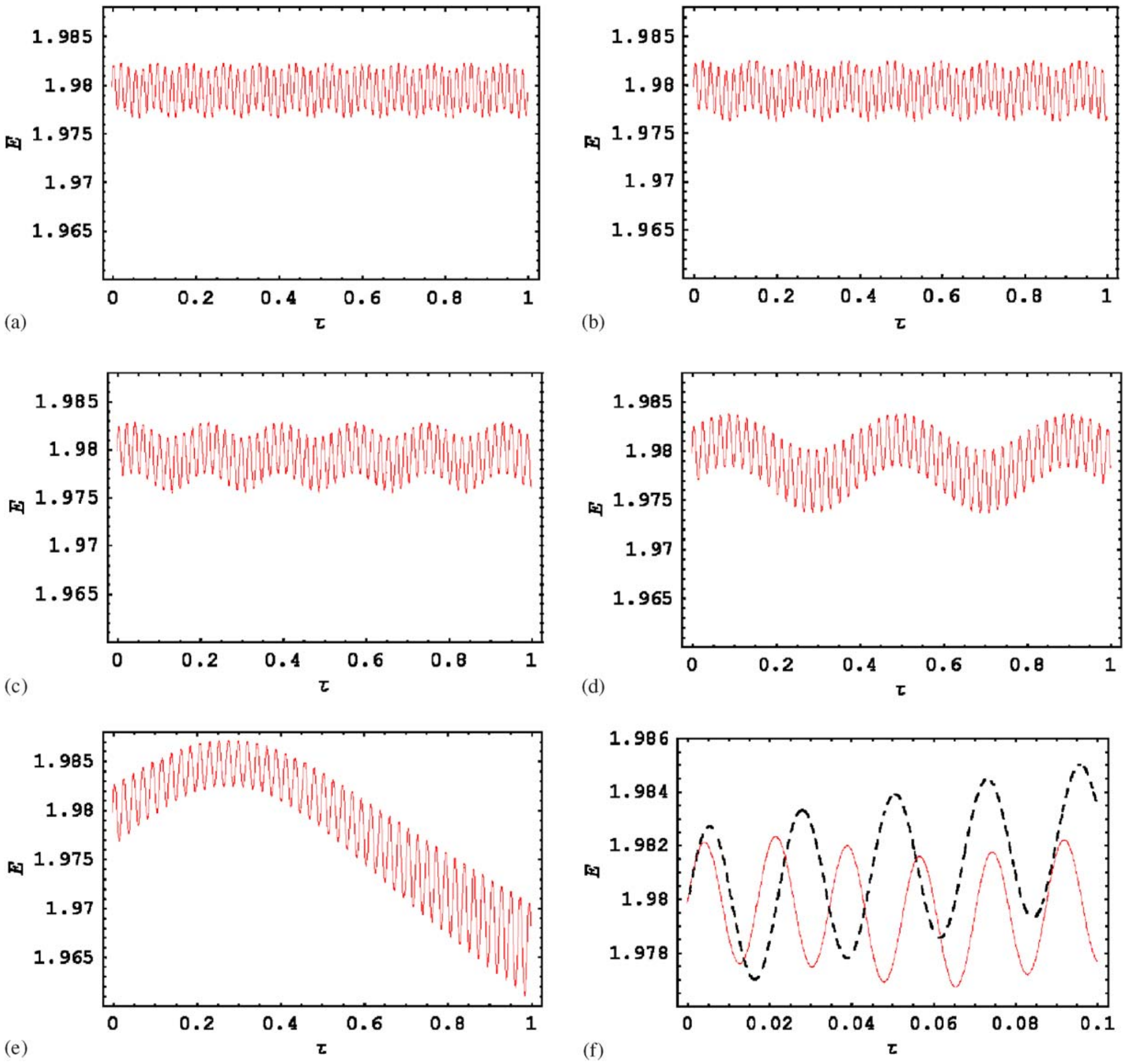

Fig. 3. Magnitude of the total magnetization, E, as function of time for $m/(d^3H_0) = 10^2$ and different values of exchange parameter: (a) $Jm/H_0 = 0$, (b) $Jm/H_0 = 5$, (c) $Jm/H_0 = 10$, (d) $Jm/H_0 = 15$ and (e) $Jm/H_0 = 20$. The Fig 3(f) corresponds to a zoom in the time interval of the Fig. 3(a) and (e), where the dashed line is for (e).

spanned by the individual particles has a hole between the initial condition points. When the Zeeman energy is the relevant term, the dynamics is quasi-periodic and the orbits in the phase space are almost contained in a plane perpendicular to the external field describing rose shape trajectories and, finally, when the all three terms are comparable we have found that, the temporal evolution of the each magnetic moment is periodic and its phase space is a ring on a sphere.

4. Conclusion

We have performed a detailed study of a magnetic dimer including dipolar and exchange interactions, in the presence of an applied magnetic field, but without dissipation. We found that, the individual magnetization components have periodic and quasi-periodic time dependence with different time scales and that the corresponding three-dimensional phase diagrams present different confinement, determined by the strength ratios of dipolar and exchange parameters. Furthermore, the magnitude of the total magnetization of this system is a fluctuating time-dependent function which is a consequence of the dipolar term, and its dynamical behavior strongly depends on the relative interaction coupling constants; also the envelope function contain the information of the exchange interaction.

Acknowledgments

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